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Research Article

NMC and the Fine-Tuning Problem on the Brane

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We propose a new solution to the fine-tuning problem related to coupling constant λ of the potential. We study a quartic potential of the form $\lambda\phi^4$ in the framework of the Randall-Sundrum type II braneworld model in the presence of a Higgs field which interacts nonminimally with gravity via a possible interaction term of the form $-(\xi/2)\phi^2 R$. Using the conformal transformation techniques, the slow-roll parameters in high energy limit are reformulated in the case of a nonminimally coupled scalar field. We show that, for some value of a coupling parameter ξ and brane tension T, we can eliminate the fine-tuning problem. Finally, we present graphically the solutions of several values of the free parameters of the model.

1. Introduction

The viability of an inflationary scenario and the constraints on the inflationary model are profoundly affected by the presence of NMC (nonminimal coupling) and by the value of the coupling constant ξ [1]. The analysis of the various inflationary scenarios considered in the literature usually leads to the result that NMC makes it harder to achieve inflation with a given potential that is known to be inflationary for $\xi = 0$.

Among the various models of the inflationary scenario, Linde's chaotic model [2] has been regarded as a feasible and natural mechanism for the realization of inflationary expansion. This model still has a serious problem; that is, one has to fine-tune the self-coupling constant λ of the inflaton which is unacceptably small to have a reasonable amplitude of the density perturbations.

On the other hand, Fakir and Unruh (FU) proposed a way to avoid fine-tuning λ by introducing a relatively large nonminimal coupling constant $|\xi| > 1$ in the context of the chaotic inflationary model. According to their results, the large value of ξ, that is, order of 10^3, allows us to have a reasonable value for the coupling constant $\lambda = 10^{-2}$. Thus, the FU scenario remains a reasonable model of the inflationary scenario [3].

Nevertheless, the feasibility of inflation has been also investigated in alternative theories of gravity, for example, the Brans-Dicke scalar tensor theory [4, 5], and nonminimal coupling theories of gravity [6]. Furthermore, the analytical study of the same brane model with a nonminimally coupled scalar has also been studied in [7, 8].

Recently there was an extensive discussion of Higgs inflation in the theory with the potential $(\lambda/4)(\phi^2 - \upsilon^2)^2$ and nonminimal coupling to gravity $(\xi/2)\phi^2 R$ [9], with the Higgs field playing the role of the inflaton field. The main idea was to introduce a large nonminimal coupling of the Higgs field to the curvature scalar, and after that, one can make a transformation from the Jordan frame to the Einstein frame to render the potential of the Higgs field sufficiently flat to support inflation. The effect of this coupling is to flatten the SM potential above the scale $(M_{\mathrm{Pl}}/\sqrt{\xi})$ thereby allowing a sufficiently flat region for slow-roll inflation [10].

There has recently been a revival of the idea that inflation based on scalar fields which are nonminimally coupled to gravity can account as a way to unify inflation with weak-scale physics [11]. However, it has been suggested that such models are unnatural, due to an apparent breakdown of the calculation of Higgs-Higgs scattering via graviton exchange in the Jordan frame [12]. On the other hand, it was shown [13] that Higgs inflation models are in fact natural and that the breakdown does not imply new physics due to strong-coupling effects or unitarity breakdown but simply a failure of perturbation theory in the Jordan frame as a calculational method. Also, it was suggested that Higgs inflation is a completely consistent and natural slow-roll inflation model when studied in the Einstein frame [13].

In addition, a new class of models of chaotic inflation in supergravity was proposed in [14]. Moreover, this new class of supergravity models can describe an inflaton field which is nonminimally coupled to gravity, with coupling $(\xi/2)\phi^2 R$ [15]. In this class, in order to make these models consistent with observations, there is no need to go to the limit $\xi \gg 1$; a very small positive value of ξ is quite sufficient.

In this work, we are interested in an inflationary model in the braneworld model with a quartic potential of the form $\lambda\phi^4$ in the Einstein frame with a nonminimally coupled Higgs scalar field $-(\xi/2)\phi^2 R$ in relation to recent observations [16]. Using the conformal transformation techniques [17], the slow-roll parameters with NMC are obtained, and we show that we can eliminate the fine-tuning problem related to the coupling constant λ of the potential for some value of a coupling parameter ξ and brane tension T. We will also show that, depending on the parameters of the model, one can cover a significant part of the space of parameters $(n_s; r; dn_s/d\ln(k))$ allowed by recent observational data.

The plan of the paper is as follows. In the next section, we recall first the conformal transformation techniques. In Section 3, we study the RS2-model with a nonminimally coupled Higgs scalar field. In Section 4, we present our results for a quartic potential in the Einstein frame on the brane. A conclusion is given in the last section.

2. Conformal Transformation Techniques

Recently, the use of conformal transformation techniques has become widespread in the literature on gravitational theories alternative to general relativity and in particular on nonminimally coupled scalar fields to the spacetime Ricci curvature scalar. As it is well known, if only one scalar field is nonminimally coupled, then one may perform a conformal transformation to a new frame in which both the gravitational portion of the Lagrangian and the kinetic term for the (rescaled) field assume canonical form [18].

Consider general scalar-tensor theories (STT), in D-dimension with the metric $g_J^{\mu\nu}$ (to be called the Jordan conformal frame), for which the gravitational field action is written in the following form [17]:

$$S = \int d^D x \sqrt{-g_J}\left[f(\phi) R_J - \frac{1}{2} g_J^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V_J(\phi)\right], \quad (1)$$

where $f(\phi)$ is positive definite. G_D is a gravitational constant in D dimensions.

The nonminimal coupling associated with the renormalization counterterm takes the following form:

$$f(\phi) = \frac{1}{2}\left[\frac{1}{8\pi G_D} - \xi\phi^2\right]. \quad (2)$$

Note that this term corresponds to a nonminimally coupled scalar field with an interaction term of the form $\pounds_{\text{int}} = -(\xi/2)\phi^2 R$, where ξ is a dimensionless coupling constant. Minimal coupling corresponds to $\xi = 0$, where

$$f(\phi) = \frac{1}{16\pi G_D}. \quad (3)$$

We may further parameterize the following, in terms of a Planck mass in D dimensions:

$$M_{(D)}^{D-2} = \frac{1}{8\pi G_D}. \quad (4)$$

For $D = 4$, we have $M_4 = M_p = 1/\sqrt{8\pi G_4} = 2.4 \times 10^{18}$ GeV. In the sign conventions of (2), a conformally coupled field has

$$\xi_c = \frac{1}{4}\left(\frac{D-2}{D-1}\right). \quad (5)$$

We may use a conformal transformation of the metric, defined as

$$\sqrt{-g_E} = \Omega^D \sqrt{-g_J}, \quad (6)$$

where

$$\Omega^{D-2} = \frac{2}{M_{(D)}^{D-2}} f(\phi), \quad (7)$$

$\sqrt{-g_E}$ is determinant of the metric in the Einstein frame, and a new field φ appears which is related to the field ϕ via [17]

$$\left(\frac{d\varphi}{d\phi}\right)^2 = \frac{M_{(D)}^{D-2}}{2f^2(\phi)}\left(f(\phi) + \frac{2(D-1)}{(D-2)}\left(\frac{df}{d\phi}\right)^2\right) = \frac{1}{Z(\phi)}. \quad (8)$$

The combination of (8) is always nonzero, because we have only considered models in which $f(\phi)$ is positive definite and real.

In terms of new variables, the action of the model is given by

$$S = \int d^D x \sqrt{-g_E}\left[\frac{M_{(D)}^{D-2}}{2} R_E - \frac{1}{2} g_E^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - V_E(\phi(\varphi))\right], \quad (9)$$

where we have introduced a transformed effective potential

$$V_E(\phi) = \frac{V_J(\phi)}{\Omega^D}. \quad (10)$$

The action equation (9) is similar to that of GR with a minimally coupled scalar field φ in addition to arbitrary D. This has also both the canonical Einstein-Hilbert form and the canonical kinetic term for the scalar field.

In the next section, we will clarify the link between the conformal transformation techniques and the slow-roll approximation. We prove that slow-roll inflation in the physical Jordan frame implies slow-roll inflation in the Einstein frame.

3. NMC and the Slow-Roll Approximation to Inflation

We start this section by recalling briefly some foundations of Randall-Sundrum type 2 braneworld model with a non-minimally coupled bulk scalar field via an interaction term of the form $-(\xi/2)\phi^2 R$. We will use here the conformal transformation techniques introduced in Section 1 in the case of $D = 5$. In this scenario, our universe is considered as a 3-brane embedded in five-dimensional anti-de Sitter space-time (AdS5), where gravitation can propagate through a supplementary dimension. One of the most relevant consequences of this model is the modification of the Friedmann equation for energy density of the order of the brane tension or higher.

The field ϕ in the Einstein frame with $\xi \neq 0$ is not canonically normalized. In effect, the canonically normalized inflaton field φ can be related, via (8), to the field ϕ as follows:

$$\left(\frac{d\varphi}{d\phi}\right)^2 = \frac{1 - \xi\phi^2/M_5^3 + (16/3)\left(\xi^2\phi^2/M_5^3\right)}{\left(1 - \xi\phi^2/M_5^3\right)^2} = \frac{1}{Z(\phi)}. \quad (11)$$

The four-dimensional Planck scale is given by [19]

$$M_p = \sqrt{\frac{3}{4\pi}}\left(\frac{M_5^3}{\sqrt{T}}\right), \quad (12)$$

where $M_p = 1.2 \times 10^{19}$ GeV.

In doing the slow-roll analysis, we should note that slow-roll approximation puts a constraint on the slope and the curvature of the potential. This is clearly seen from the field expressions of ϵ and η parameters. Thus, in the Einstein frame, it is φ and not ϕ that has a canonical kinetic term. Therefore, the slow-roll parameters are

$$\epsilon = \frac{M_p^2 T V_\varphi'^2}{4\pi V^3}, \qquad \eta = \frac{M_p^2 T V_\varphi''}{4\pi V^2}, \quad (13)$$

where $V_\varphi'' = d^2V/d\varphi^2$ and T is the brane tension. We signal that the slow-roll approximation takes place if these parameters are such that $\max\{\epsilon, |\eta|\} \ll 1$ and inflationary phase ends when ϵ or $|\eta|$ are equal to one. Before proceeding, it is interesting to comment low and high energy limits of these parameters. Note that at low energies where $V \ll T$, the slow-roll parameters take the standard form. At high energies (our case) $V \gg T$, the extra contribution to the Hubble expansion dominates.

Another important characteristic inflationary parameter is the number of e-folding N defined by [19]

$$N \simeq -\frac{4\pi}{TM_p^2}\int_{\varphi_*}^{\varphi_{\text{end}}} \frac{V^2}{V_\varphi'} d\varphi. \quad (14)$$

The small quantum fluctuations in the scalar field lead to fluctuations in the energy density and in the metric. For these reasons, we define the power spectrum of the curvature perturbations by [20]

$$P_R(k) \simeq \frac{16\pi V^6}{3M_p^6 T^3 V_\varphi'^2}. \quad (15)$$

The ratio of tensor to scalar perturbations r is

$$r \simeq 24\epsilon. \quad (16)$$

In relation to $P_R(k)$, the scalar spectral index is defined as [20]

$$n_s - 1 = \frac{d\ln P_R(k)}{d\ln k}, \qquad \simeq -6\epsilon + 2\eta. \quad (17)$$

Another perturbation parameter spectrum quantity is the running of the scalar index $dn_s/d\ln k$, defined as

$$\frac{dn_s}{d\ln k} \simeq \frac{M_p^2 V_\varphi' T}{2\pi V^2}\left(3\frac{\partial\epsilon}{\partial\varphi} - \frac{\partial\eta}{\partial\varphi}\right). \quad (18)$$

From now on, we set $V = V_E$ and we use (11) to calculate the slow-roll parameters as function of ϕ as follows:

$$\epsilon = Z\epsilon_\phi, \quad (19)$$

where $\epsilon_\phi = M_p^2 T V_\phi'^2/4\pi V^3$ and

$$\eta = Z\eta_\phi + Z'\frac{M_p^2 T}{8\pi}\frac{V_\phi'}{V^2}, \quad (20)$$

where $Z' = dZ/d\phi$ and $\eta_\phi = M_p^2 T V_\phi''/4\pi V^2$. This does not change the direction of rolling but modifies the velocity of the field such that with heavier rolling mass the motion becomes slower (smaller ϵ and η) as indicated by (19) and (20). Similarly, we can find the number of e-folds by

$$N \simeq -\frac{4\pi}{TM_p^2}\int_{\phi_*}^{\phi_{\text{end}}} \frac{V^2}{ZV_\phi'} d\phi, \quad (21)$$

where the subscripts $*$ and end are used to denote the epoch when the cosmological scales exit the horizon and the end of inflation, respectively.

The power spectrum of the curvature perturbations and the running of the scalar index becomes, respectively,

$$P_R(k) \simeq \frac{16\pi V^6}{3M_p^6 T^3 Z V_\phi'^2}, \qquad \frac{dn_s}{d\ln k} \simeq \frac{M_p^2 TZV_\phi'}{2\pi V^2}\left(3\frac{\partial\epsilon}{\partial\phi} - \frac{\partial\eta}{\partial\phi}\right). \quad (22)$$

In what follows, we will apply the above braneworld formalism with a quadratic potential in the Einstein frame to derive perturbation spectrum in relation to recent Planck data [16]. This potential has been recently studied in [15] for the standard inflation.

Table 1

Braneworld inflationary parameters	λ	n_s	r	$\frac{dn_s}{dlnk}$
$N = 50$	$\simeq 2.57 \times 10^{-15}$	0.940	0.315	−0.00116
$N = 60$	$\simeq 1.49 \times 10^{-15}$	0.950	0.263	−0.000815
$N = 80$	$\simeq 6.37 \times 10^{-16}$	0.962	0.198	−0.000461
$N = 100$	$\simeq 3.28 \times 10^{-16}$	0.970	0.158	−0.000296
Combination of Planck + WP data		0.9603 ± 0.0073	$r < 0.11$	-0.0134 ± 0.0090

4. Perturbation Spectrum for a Quartic Potential

The coupling constant ξ is often regarded as a free parameter in inflationary scenarios. This view arises from the fact that there is no universal prescription for the value of ξ. Indeed, some prescriptions for ξ do exist in specific theories, although they are not widely known and they depend on the nature of the scalar field ϕ and on the theory of gravity [21].

On the other hand, the prevailing point of view in the literature on inflation is that the coupling constant is a free parameter and that the values of that which are acceptable are those that, a posteriori, make a specific inflationary scenario viable [22]. Thus, in the context of cosmology, it has been recognized that nonminimal fields could be employed to solve some problems associated with inflation, for example, the fine-tuning problem related to coupling constant λ. Note that more inflationary scenarios have studied this problem; for another type of models, see [23, 24].

We will consider the case of a quartic potential, in the Jordan frame, $V_J = \lambda\phi^4$, which leads to the Einstein frame potential as follows:

$$V_E = \frac{\lambda\phi^4}{\left(1 - \xi\phi^2/M_5^3\right)^{5/3}}. \quad (23)$$

The model with $\xi = 0$, which is equivalent to considering the inflaton Higgs field to be minimally coupled to gravity, is singled out by its simplicity and the potential $V(\phi)$ is very flat. In this context, one can also consider the slow-roll parameters to study the spectrum of the perturbation. The two first parameters are given for this model by

$$\begin{aligned}\epsilon &= \frac{TM_p^2}{9\pi\lambda M_5^6}\left(1 - \frac{\xi\phi^2}{M_5^3}\right)^{5/3}\left(6M_5^3\left(1 - \frac{\xi\phi^2}{M_5^3}\right) + 5\xi\phi^2\right)^2 \\ &\quad \times \left(\phi^6\left(1 - \frac{\xi\phi^2}{M_5^3} + \frac{16\xi^2\phi^2}{3M_5^3}\right)\right)^{-1}, \\ \eta &= \frac{TM_p^2}{12\pi\lambda M_5^6}\frac{\left(1 - \xi\phi^2/M_5^3\right)^{5/3} A}{\phi^6\left(1 - \xi\phi^2/M_5^3 + 16\xi^2\phi^2/3M_5^3\right)},\end{aligned} \quad (24)$$

where

$$\begin{aligned}A &= 324M_5^6\phi^2 - 684M_5^3\xi\phi^4 + 324\xi^2\phi^6 \\ &\quad + 810M_5^6\xi\phi^4 - 810M_5^6\xi^2\phi^6 + 18M_5^3\xi + 9\xi^2\phi^2 \\ &\quad - 27\xi^3\phi^4 - 32\xi^2M_5^3 + 48\xi^3\phi^2.\end{aligned} \quad (25)$$

On the other hand, one can derive the number of e-folding as

$$N \simeq -\frac{\pi\lambda}{TM_p^2}\int_{\phi_*}^{\phi_{\text{end}}} \frac{\phi^5\left(1 - \xi\phi^2/M_5^3 + 16\xi^2\phi^2/3M_5^3\right)}{\left(1 - \xi\phi^2/M_5^3\right)^{8/3}\left(1 + 3\left(\xi\phi^5/M_5^3\right)\right)} d\phi. \quad (26)$$

Although we have analytic results for slow-roll parameters, it is not easy to solve them to obtain ϕ_* at which the observables n_s, r, and $dn_s/d\ln k$ should be evaluated. Instead, we proceed numerically by finding ϕ_{end} and then going $N = 60$ e-folds back to obtain ϕ_* while making sure that the slow-roll parameters remain small in this range of ϕ.

Moreover, the combination of Planck + WP data, with the pivot scale chosen at $k_0 = 0.05\,\text{Mpc}^{-1}$, gives the following results [16]:

$$n_s = 0.9603 \pm 0.0073 \quad (68\%\ \text{CL}), \quad (27)$$

$$r < 0.11 \quad (95\%\ \text{CL}), \quad (28)$$

$$\frac{dn_s}{dlnk} = -0.0134 \pm 0.0090 \quad (68\%\ \text{CL}), \quad (29)$$

$$P_R(k) = 2.215 \times 10^{-9} \quad (68\%\ \text{CL}). \quad (30)$$

In the following, we will study different regimes of coupling ξ. Our results will be compared to observations and the coupling effect on perturbation inflationary spectrum will be studied.

4.1. Minimal Coupling Case ($\xi = 0$). We investigate in this case the simplest chaotic potential of the form $V_J = \lambda\phi^4$ in the Jordan frame with a minimally coupled scalar field ($\xi = 0$). Equation (26) gives the integrated expansion from ϕ_* to ϕ_{end} as

$$N = \frac{1}{6}\frac{\pi\lambda}{M_p^2 T}\left(\phi_*^6 - \phi_{\text{end}}^6\right). \quad (31)$$

After that, we calculate the value of the scalar field at the end of inflation, in terms of λ and T. Using this, (31) for N can be solved to give $\phi_N(\lambda, T)$, where ϕ_N is the value of the scalar field e-foldings before the end of inflation. Finally, starting from (30), we can plot the variations of physical quantities for various values of N (see Table 1).

We remark that, in Table 1, for different values of N, we obtain the very small values of the coupling constant λ which introduces the existence of the fine-tuning problem. These results coincide with the results of the chaotic inflation in

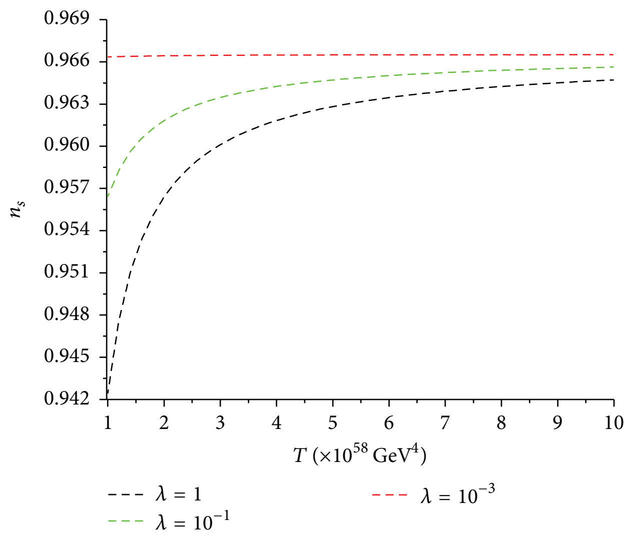

Figure 1: n_s versus T for different values of λ for $\xi_c = 3/16$ and $N = 60$.

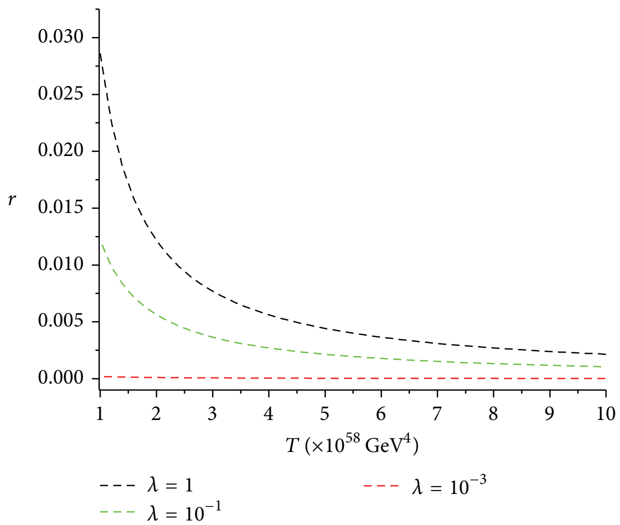

Figure 2: r versus T for different values of λ for $\xi_c = 3/16$ and $N = 60$.

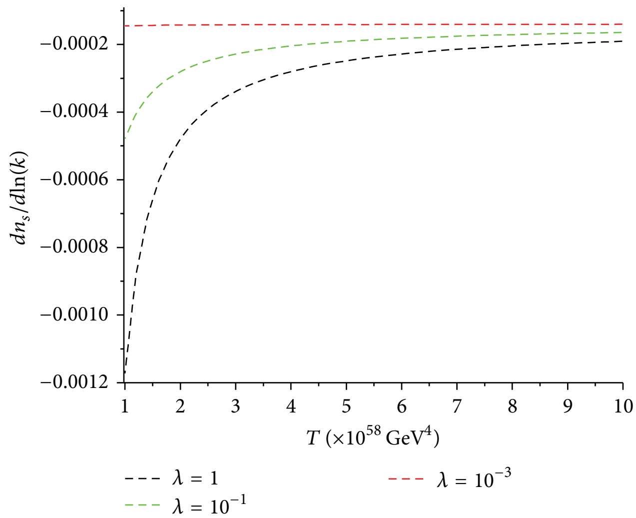

Figure 3: $dn_s/d\ln(k)$ versus T for different values of λ for $\xi_c = 3/16$ and $N = 60$.

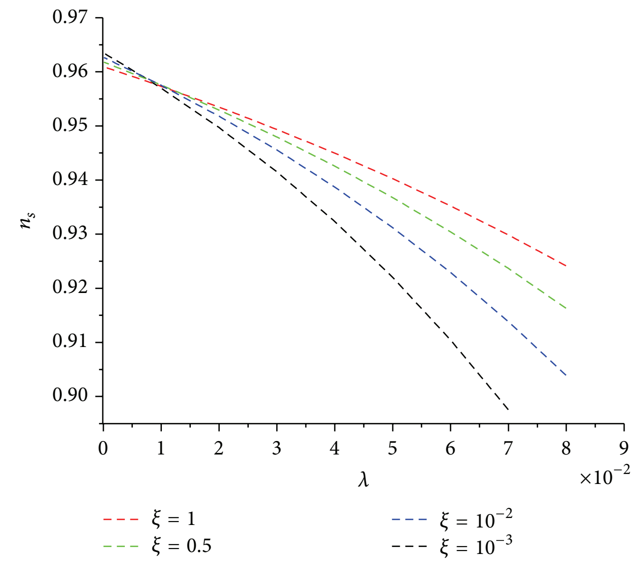

Figure 4: n_s versus λ for different values of ξ for $T = 5 \times 10^{58}$ GeV^4 and $N = 60$.

the standard case. We remark also that the parameters n_s and r coincide with the observation values except for $N > 60$. Note that, in this case, it does not show the effect of the brane tension T because it is simplified in the calculation.

In what follows, we will show that the addition of the parameter ξ (where $\xi \neq 0$) and the brane tension T has a significant impact on the spectrum of the model and gives solutions to the fine-tuning problem and the inflation can occur successfully.

4.2. Conformal Coupling Case ($\xi_c = 3/16$). In this section, we limit the discussion to the model when $\xi = \xi_c$ (ξ_c is the five-dimensional conformal coupling) where we use the potential in the Einstein frame of the form $V_E = \lambda\phi^4/(1 - \xi\phi^2/M_5^3)^{5/3}$.

Figure 1 shows the variation of the scalar spectral index n_s as function of T for different values of the coupling constant λ. We observe that values of T of about $\approx 10^{58}$ GeV^4 may have a cover n_s interval which coincides with the observation data for $\lambda = 1$, where the fine-tuning problem is solved. We also note that, for small values of $\lambda \preccurlyeq 10^{-3}$, $n_s < 0.97$ for $T \succcurlyeq O(10^{58}$ GeV$^4)$.

Figure 2 shows the variation of the ratio r with respect to T for different values of the coupling constant λ. We show that r is a decrease function with T for the different values of λ. We have also shown that r is consistent with the observation data for $\lambda \succcurlyeq O(10^{-2})$. We emphasize that, for small values of $\lambda \preccurlyeq 10^{-3}$, the r is negligible.

Figure 3 shows the variation of the running of the scalar spectral index $dn_s/d\ln k$ as function of T for different values

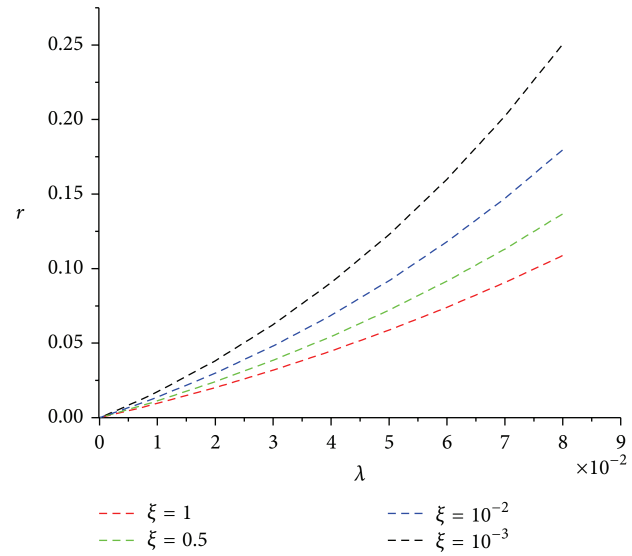

FIGURE 5: r versus λ for different values of ξ for $T = 5 \times 10^{58}$ GeV^4 and $N = 60$.

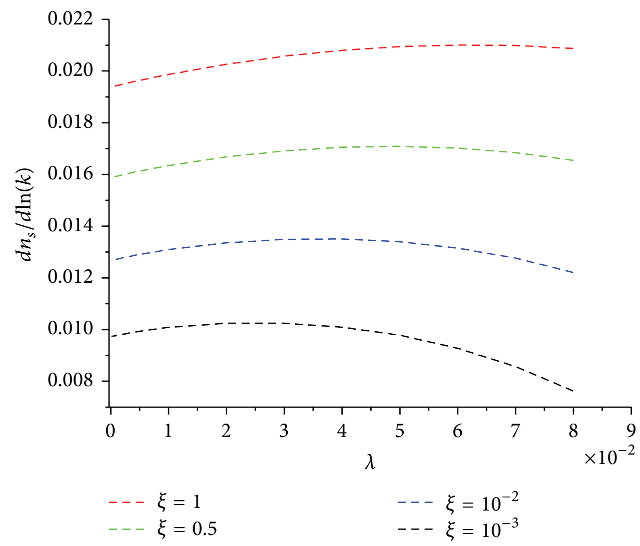

FIGURE 6: $dn_s/d\ln(k)$ versus λ for different values of ξ for $T = 5 \times 10^{58}$ GeV^4 and $N = 60$.

of the coupling constant λ. We show that the running of the scalar spectral index $dn_s/d\ln k$ is small and negative which is consistent with recent observation data. We also remark that, for small values of $\lambda \preccurlyeq 10^{-3}$, the $dn_s/d\ln k$ does not exceed 0 ($dn_s/d\ln k \lesssim 0$).

Note that, in order to obtain the value of the power spectrum of the curvature perturbations $P_R(k) \simeq 2.215 \times 10^{-9}$ for $\xi_c = 3/16$, one should take the following values $T = 6.3 \times 10^{58}$ GeV^4, $\lambda = 1$, and $N = 60$.

Finally, in this case, we have shown that by variation of the free parameters λ and T, the results reviewed by the value of ξ_c are compatible with the observational data for the value of $\lambda = 1$ where the fine-tuning problem is solved and the inflation can occur successfully. For example, for $N = 60$, $T = 6.3 \times 10^{58}$ GeV^4, and $\lambda = 1$, we find the central value $n_s \simeq 0.961$, $r \simeq 0.0036$, and $dn_s/d\ln k \simeq -2.22 \times 10^{-4}$.

4.3. General Case. In this case, we will discuss possible physical implications of the solutions in the case of $0 \prec \xi \prec \xi_c$ and $\xi_c \prec \xi \preccurlyeq 1$. The motivation is to examine whether this model coincides with the observation for $\xi \neq \xi_c$ and $\xi > 0$.

In Figure 4, we present the variations of scalar spectral index n_s according to the value of the coupling constant λ for different values of ξ. We show that n_s in the two regions decreases when λ increases and we can have a range of n_s for small values of λ where the four curves are almost confounded which is in agreement with the observations.

Figure 5 shows the variation of the ratio r with respect to λ for different values of the coupling constants ξ. We can remark in Figure 5 that the ratio r increases with respect to λ for two regions $0 \prec \xi \prec \xi_c$ and $\xi_c \prec \xi \preccurlyeq 1$ which is in agreement with recent observation data. On the other hand, for values of λ of the order of $O(0.1 - 1)$, $r \succ 0.24$ which is outside the observed region.

In Figure 6, we present the variations of running of the scalar spectral index $dn_s/d\ln k$ according to the value of the coupling constant λ for different values of ξ. We emphasize that $dn_s/d\ln k$ is positive and not consistent with the observation for different values of ξ and λ; see (29).

In summary, we note that, in this section, with potential of the form $V(\phi) = \lambda\phi^4$, we find a more precise interval of the nonminimal coupling for assisted inflation and the parameters n_s and r are in agreement with observation for the free parameters of the model.

5. Conclusion

In this paper, we have studied a quartic potential of the form $\lambda\phi^4$ in the Einstein frame with the Higgs field ϕ nonminimally coupled to gravity for the different cases of ξ and for arbitrary values of the parameters T and λ. The slow-roll parameters in the brane are reformulated in the case of a nonminimally coupled scalar field. We have shown that, by changing parameters of these models, one can cover a significant part of the space of the observable parameters (n_s; r; $dn_s/d\ln k$) allowed by recent observational data for the different cases of ξ. We have also shown that, in order to avoid fine-tuning λ of the potential, the introducing of a relatively nonminimal coupling constant ξ of the order of unity is quite sufficient to allow us to have a reasonable value for the coupling constant $\lambda \geq 10^{-2}$ for $T = O(10^{58}$ GeV$^4)$. In addition, we have examined the properties of the new case. The case of $\xi < 0$ predicts undesirable values of (n_s; r; $dn_s/d\ln k$) with recent observational data. Finally, there has been a great deal of work on NMC in the inflationary models; this paper justifies certain methods used, solves some of the problems posed, and proves that inflation, with nonminimal coupling, can occur successfully for the simplest inflationary models in braneworld.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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